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Finite volume method for analysis of stress and strain in wood

Metoda konačnih volumena za analizu naprezanja i deformacija u drvu

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ABSTRACT • *This paper presents a numerical method (the finite volume method) for analysing stress and strain in wood as a solid body. The method is very simple and easy to use. It starts from an integral form of the equations governing momentum, heat and mass balance. Second-order in both time and space finite volume discretisation is performed using the corresponding constitutive relations, resulting in a set of algebraic equations, which are then solved by an efficient segregated iterative procedure. In order to demonstrate the method's possibilities, stress and deformation are analysed in a loaded chair and in wood samples during the process of wood drying and steaming.*

Key words: *finite volume method, chair, wood, drying, steaming*

SAŽETAK • *U radu je prikazana numerička metoda (metoda konačnih volumena) za analizu naprezanja i deformacija u drvu kao čvrstom tijelu. Metoda polazi od integralnog oblika jednadžbi bilance količine gibanja, topline i mase. Da bi se taj sustav jednadžbi zatvorio, primijenjene su konstitutivne relacije, a zatim su definirani početni i rubni uvjeti. Nakon toga provedeno je diskretiziranje drugog reda točnosti po vremenu i prostoru. Dobiven je sustav nelinearnih, povezanih algebarskih jednadžbi, koji je riješen efikasnom iterativnom metodom.*
Da bi se demonstrirala mogućnost primjene metode, urađeni su primjeri iz primarne i finalne obrade drva. U prvom su primjeru analizirani naprezanje i deformacije u opterećenoj stolici. Drugi primjer obrađuje sušenje drvenih gredica. Za vrijeme procesa sušenja izračunani su temperatura, koncentracija vlage, deformacije i naprezanja u ortotropnom materijalu, čija su se fizikalna svojstva mijenjala s promjenom temperature i vlažnosti. Treći je primjer proračun temperature, deformacija i naprezanja u drvu tijekom parenja. Materijal je tretiran kao termo-elastoplastičan.

Ključne riječi: *metoda konačnih volumena, stolica, drvo, sušenje, parenje*

1 INTRODUCTION
1. UVOD

In the past development of new products in the wood industry, new drying schedules or new wood steaming schedules were mostly based on practice. The development of computer technology and numerical methods have made the research much easier and enabled obtaining information of what is happening inside a loaded product. This paper presents the finite volume method. This method for stress analysis is equally applicable to linear, isotropic, anisotropic, porous and non-linear materials. In what follows, an outline of the method is given and some results illustrating the method's possibilities are presented. More details about the method can be found in previous works (Demirdžić and Martinović, 1993; Demirdžić et al, 2000; Martinović *et al*, 2001; Horman *et al*, 2003; Hajdarević *et al*, 2006).

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2 MATHEMATICAL MODEL
2. MATEMATIČKI MODEL

The behaviour of an arbitrary part of a solid, porous body of volume V bounded by the surface S at any instant of time t can be described by equations of momentum, thermal energy and mass balance (Demirdžić *et al*, 2000)

$$\frac{\partial}{\partial t}\int_V \rho \frac{\partial u_i}{\partial t} dV = \int_S \rho_{ij} n_j dS + \int_V f_i dV \tag{1}$$

$$\frac{\partial}{\partial t}\int_V \rho c_q T dV = -\int_S q_j n_j dS + \int_V s_q dV \tag{2}$$

$$\frac{\partial}{\partial t}\int_V \rho c_m M dV = -\int_S m_j n_j dS + \int_V s_m dV \tag{3}$$

where ρ is the density, u_i is the displacement, σ_{ij} is the stress tensor, f_i is the volume force, q_j and m_j are the heat and mass flux vector, T is the temperature, M is the moisture potential, c_q and c_m are the specific heat and specific moisture, S_q and S_m are the heat and mass source, n_j is the outwarded unit normal to the surface S.

In order to close the system of Eqs. (1) to (3) or (1) and (2) or (1), the constitutive relations are used:
– for an elastic, porous, orthotropic material
- for Eq. (1):

$$\begin{bmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{12}\\ \sigma_{23}\\ \sigma_{31}\end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} & 0 & 0 & 0\\ A_{21} & A_{22} & A_{23} & 0 & 0 & 0\\ A_{31} & A_{32} & A_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & A_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & A_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & A_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_{11} - \alpha_{11}\Delta T - \langle \beta_{11}\Delta M\rangle\\ \varepsilon_{22} - \alpha_{22}\Delta T - \langle \beta_{22}\Delta M\rangle\\ \varepsilon_{33} - \alpha_{33}\Delta T - \langle \beta_{33}\Delta M\rangle\\ \varepsilon_{12}\\ \varepsilon_{23}\\ \varepsilon_{31}\end{bmatrix} \tag{4}$$

where ε_{ij} is the strain tensor, and the nine non-zero orthotropic elastic constants A_{ij} are related to the Young's moduli E_i, the Poisson's coefficients v_{ij} and the shear moduli G_{ij} (Bodig and Jayne, 1993), α_{ij} are the coefficients of thermal expansion, β_{ij} are the shrinkage (contraction) coefficients, $\Delta T = T - T_u$, $\Delta M = M - M_h$ and T_u is the temperature at an undeformed state and M_h is the moisture potential at the fiber saturation point. The terms in $\langle\ \rangle$ brackets are "active" only for $M \langle M_h$.

- for Eqs. (2) and (3):

$$q_j = -k_{jl}^q \frac{\partial T}{\partial x_l} + \varepsilon r m_j = -(k_{jl}^q + \varepsilon r \delta k_{jl}^m) - \varepsilon r k_{jl}^m \frac{\partial M}{\partial x_1} \tag{5}$$

$$m_j = -k_{jl}^m \frac{\partial M}{\partial x_l} - \delta k_{jl}^m \frac{\partial T}{\partial x_1} \tag{6}$$

where k_{jl}^q and k_{jl}^m are the heat and mass conduction coefficient tensor components, ε is the ratio of the vapour diffusion coefficient to the coefficient of total diffusion of moisture, r is the heat of the phase change, δ is the temperature gradient coefficient.
– for a thermo-elasto-plastic isotropic material (Mendelson, 1968)
- for Eqs. (1) and (2):

$$\delta q_{ij} = 2G\delta\varepsilon_{ij} + \lambda\delta_{ij}\delta\varepsilon_{kk} - (3\lambda + 2G)\alpha\delta_{ij}\delta T - \langle \frac{3G\sigma_{ij}^d\sigma_{kl}^d\delta\varepsilon_{kl}}{\bar{\sigma}^2\left(\frac{H'}{3G}+1\right)}\rangle \tag{7}$$

where

$$\sigma_{ij}^d = \sigma_{ij} - \frac{1}{3}\delta_{ij}\sigma_{kk} \tag{8}$$

is the stress deviator and

$$\bar{\sigma} = \left(\frac{3}{2}\sigma_{ij}^d\sigma_{ij}^d\right)^{\frac{1}{2}} \tag{9}$$

is the effective stress (in the case of Von Mises yield criterion), and Lame's constants are

$$\lambda = \frac{vE}{(1+v)(1-2v)},\ G = \frac{E}{2(1+v)} \tag{10}$$

H' is the plastic modulus, and δ_{ij} is the Kronecker delta.

In the case of elastic conditions, the expression within the brackets $\langle\rangle$ vanishes, and the constitutive relation (7) reduces to the Duhamel-Neumann form of Hooke's law.

By introducing corresponding constitutive relations into governing equations a closed system is obtained of 2 or 3 (generally non-linear and coupled) equations with two or three unknown functions of spatial coordinates and time (u_i, T, M *or* u_i, T). To complete the mathematical model, initial and boundary conditions have to be specified. Displacements, temperature and moisture potential in the whole solution domain at the initial instant of time have to be given as initial conditions. The boundary conditions ought to be specified at all boundaries in terms of displacements and/or stresses, temperature and/or heat flux and moisture potential and/or mass flux.

3 NUMERICAL SOLUTION PROCEDURE
3. NUMERIČKI POSTUPAK

The solution domain is discretised by a finite number of contiguous hexahedral control volumes (*CV*) or cells of the volume *V*, which are bounded by six cell faces of the area S_j with calculation points *P* in the *CV'*s centres (Figure 1).

The time domain is subdivided into a number of time intervals δt.

Equations (1), (2) and (3) are integrated over time interval δt and over each control volume resulting in a system of 5*N* (generally non-linear and coupled) algebraic equations of the form

$$a_{P_o}\varphi_{iP_o} - \sum_{j=1}^{n} a_{P_j}\varphi_{iP_j} = b_{\varphi_i} \tag{11}$$

where φ stands for displacement components u_i (i=1, 2, 3) or temperature T or moisture potential M, n is the number of cell faces of a control volume (Fig.1).

Figure 1 A typical control volume
Slika 1. Tipični kontrolni obujam

Systems of algebraic equations (11) are solved by an iterative procedure.

4 APPLICATION OF THE METHOD
4. PRIMJENA METODE

The method described in the previous sections has been applied to a number of both isotropic and orthotropic body deformation problems (Demirdžić and Martinović, 1993; Demirdžić *et al*, 2000; Martinović *et al*, 2001; Horman *et al*, 2003; Hajdarević *et al*, 2006).

4.1 Numerical analysis of stress and strain in a wooden chair
4.1. Numerička analiza naprezanja i deformacija u drvenoj stolici

Due to symmetry only half a chair presented in Figure 2 is analysed. The mass load of the horizontal lower skeleton of the entire chair is 100 kg. The vertical frame mass load is 22 kg. The other surfaces are unloaded. The chair is assumed to be fixed to the ground, i.e. the displacement in those points equals zero.

The chair is made of spruce. Its mechanical properties at the temperature of 20 °C, with the density of $\rho = 440$ kg/m^3 and with the moisture content of 9.8% are given in Table 1. Orthotropy of the wood material is accounted for by approximating it with an isotropic material whose elastic modulus and Poisson's ratio are calculated by employing the least-square method (Hajdarević *et al*, 2006). The corresponding Young's modulus is $E = 3.98$ GPa, while the Poisson's ratio is $\nu = 0.192$. The equation of momentum balance (1) in the static equilibrium is used as well as the constitutive relation for the elastic, isotropic material (the Duhamel-Neumann form).

The distribution of the dominant normal stress σ_{yy} on the chair skeleton surface and joints is shown in Figure 3 (left). The maximum value of this stress is 10.7 MPa, both in the tensile and compression zone and it occurs in the joint of the side rail and the back leg.

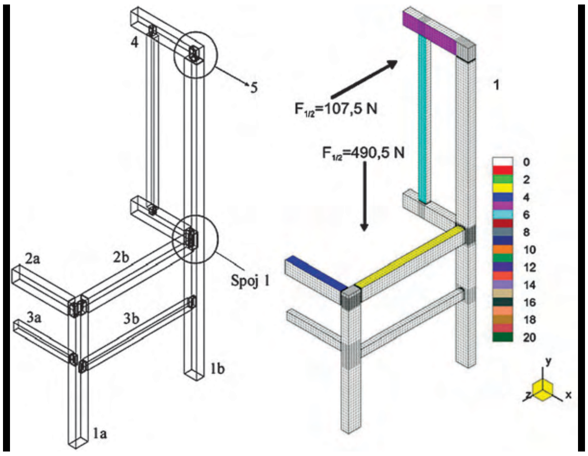


Figure 2 Chair skeleton construction (left), solution domain and numerical network (right)
Slika 2. Skeletna konstrukcija stolice (lijevo), domena rješavanja i numerička mreža (desno)

Maximum shear stress values occur at the same place. The distribution of the total shear stress on the tenon surface, left adhesive pointing zone, is shown in Figure 3 (right). The maximum total shear stress value is about 8.2 MPa and it occurs at about 2.5% of the total surface.

The distribution of the effective stress σ_{eff} on the chair skeleton surface and joints is shown in Figure 4 (left). The maximum value of this stress is ~14 MPa.

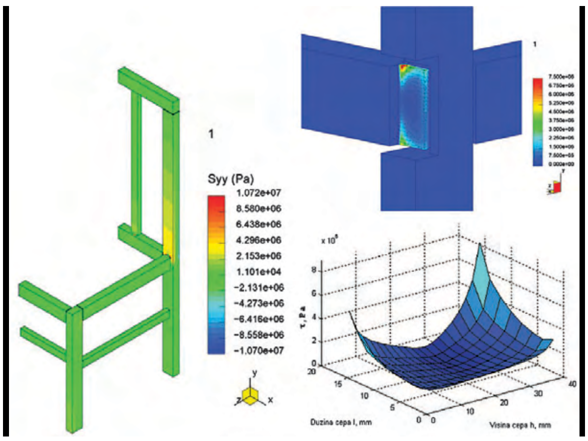


Figure 3 Distribution of stress σ_{yy} on the chair contour (left), and distribution of the total shear stress in the adhesive pointing zone at the joint between the side rail and the back leg (right)
Slika 3. Raspodjela naprezanja σ_{yy} na konturi stolice (lijevo), i raspodjela ukupnoga posmičnog naprezanja u prikazanoj adhezivnoj zoni spoja postranične spojnice i stražnje noge (desno)

Table 1 Mechanical properties of wood (spruce) (Bodig and Jayne, 1993)
Tablica 1. Mehanička svojstva drva (smrekovine) (Bodig i Jayne, 1993)

E_t	E_r	E_l	G_{rt}	G_{lr}	G_{lt}	ν_{tr}	ν_{rt}	ν_{rl}	ν_{lr}	ν_{tl}	ν_{lt}
GPa	GPa	GPa	GPa	GPa	GPa	-	-	-	-	-	-
0.392	0.686	15.916	0.0392	0.618	0.765	0.24	0.42	0.019	0.43	0.013	0.53

E – elastic modulus (*modul elastičnosti*); *G* – shear modulus (*modul smicanja*); ν – Poisson's ratio (*Poissonov koeficijent*); *t* – tangential (*tangencijalni*); *r* – radial (*radijalni*); *l* – longitudinal (*longitudinalni*)

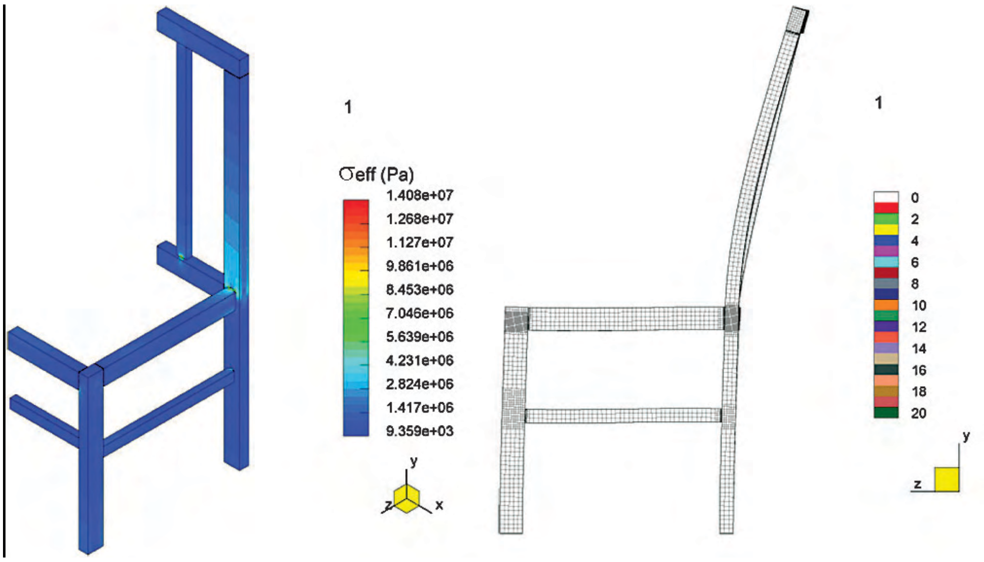


Figure 4 Distribution of stress σ_{eff} on the chair contour (left), and the strained chair skeleton (10×increase, right)
Slika 4. Raspodjela naprezanja σ_{eff} na konturi stolice (lijevo), i deformirani skelet stolice (povećanje 10 puta, desno)

The chair deformation is shown in Figure 4 (right). The largest displacement of around 13 mm occurs at the far end points of the chair back (Hajdarević *et al*, 2006).

4.2 Numerical analysis of a wood drying process
4.2. Numerička analiza procesa sušenja drva

50×50 mm thick, 600 mm long beech-wood beams are exposed to the (uniform, unsteady) flow of hot air in a laboratory dryer with an automatic control of the ambient air parameters (Fig. 5).

The temperature and moisture dependent physical properties of wood are used (Table 2). The others are considered constant and are given in Table 3.

Equations (1) to (3) and the constitutive relations (4) to (6) are used. Due to the double symmetry, only one quarter of the cross-section is taken as the solution domain. Fig. 6 shows the fields of temperature, moisture, stress σ_{xx} and displacements at t = 111 h, when the maximum stresses occur.

At this stage of the drying process the temperature field (Fig. 6a) is nearly uniform, while the moisture gradients (Fig. 6b) are still significant, causing the contraction of the wood sample (Fig. 6d), with the largest displacements in the region of the lowest moisture content. Figure 6c shows that the maximum normal stresses occur in regions near the sample's surface (Demirdžić et al, 2000).

4.3 Numerical analysis of wood heat treatment
4.3. Numerička analiza procesa toplinske obrade drva

The log is exposed to the (unsteady) flow of steam (Fig. 7) during its thermal preparation in veneer production.

The material is beech. Table 4 shows its physical properties at the temperature of 80°C and moisture content of 70%.

Equations (1) and (2) and the constitutive relations (7) and (5) (for m_j = 0) are used. The problem is considered to be a 2D plane strain problem. Fig. 8 shows the temperature distribution and the circular stress distribution at φ = *const.* and in four time instants.

The temperature gradients are the largest in the region near the log's surface and this is the region of the largest stress (Fig.8), where the residual stress occurs (Horman et al, 2003).

Figure 5 Physical domain (left), and temperature, ambient moisture and equilibrium moisture content (right)
Slika 5. Fizikalni prikaz (lijevo) i promjena temperature, relativne vlažnosti okoline i ravnotežne vlage (desno)

Table 2 Temperature and/or moisture dependent physical properties of wood (beech) (Martinović *et al*, 2001)
Tablica 2. Fizikalna svojstva drva (bukovine) ovisna o temperaturi i/ili sadržaju vode (Martinović *et al*, 2001)

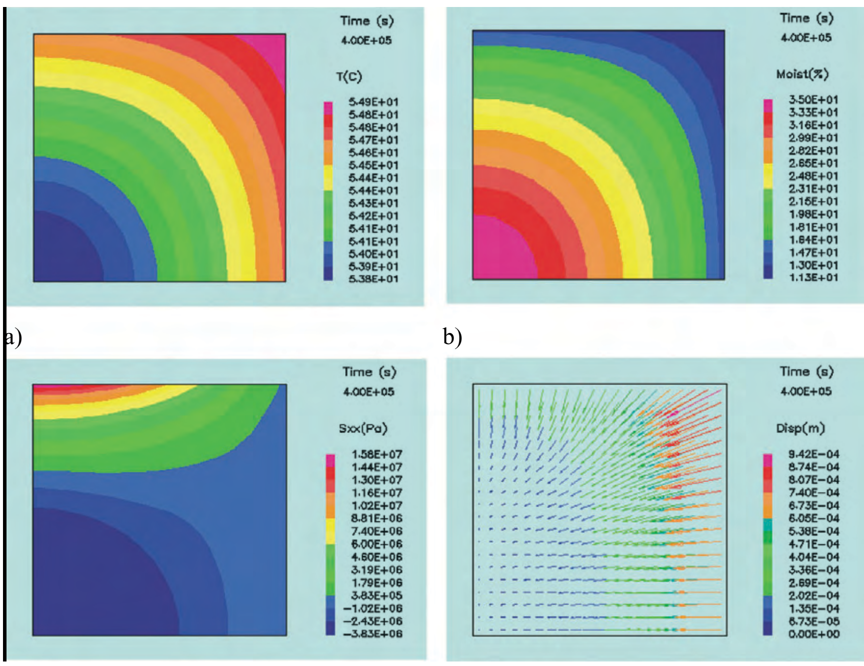
Property / *Svojstvo*	$C < 30\%$	$C \geq 30\%$
E_1(Pa)	$(6.69-4.66e^{-1,1\cdot 10^7 C^{-6,30}})(1.8-0.02T)10^8$	$2.05(1.8-0.02T)10^8$
E_2(Pa)	$(13.22-9.30e^{-2,5\cdot 10^6 C^{-5,75}})(1.8-0.02T)10^8$	$4.04(1.8-0.02T)10^8$
E_3(Pa)	$(81.11-57.0e^{-2,5\cdot 10^6 C^{-5,75}})(1.8-0.02T)10^8$	$24.8(1.8-0.02T)10^8$
ρ (kg/m^3)	$\dfrac{559(100+C)}{100-0.47(30-C)}$	$559\left(1-\dfrac{C}{100}\right)$
c_q(J/kg K)	$1173[C(100+T)]^{0,2}$	
k_{11}^q(W/m K)	$1.36(0.088+0.000709T+0.00181C)$	
k_{22}^q(W/m K)	$1.15k_{11}^q$	

$C = c_m M$ – moisture content (*sadržaj vode u drvu*); c_m – specific moisture (*specifični sadržaj vode*); M – moisture potential (*potencijal sadržaja vode*); T – temperature (*temperatura*); E – elastic modulus (*modul elastičnosti*); ρ – density (*gustoća*); c_q – specific heat (*specifični toplinski kapacitet*); k_{jl}^q – heat conduction coefficient (*koeficijent toplinske vodljivosti*)

Table 3 Constant physical properties of wood (beech)
Tablica 3. Konstantna fizikalna svojstva drva (bukovina)

Property *Svojstvo*	**Value** *Vrijednost*	**Property** *Svojstvo*	**Value** *Vrijednost*	**Property** *Svojstvo*	**Value** *Vrijednost*
r(J/kg)	$2.3\cdot 10^6$	ν_{12}	0.36	α_{11}(1/K)	$36.6\cdot 10^{-6}$
c_m(kg$_m$/kg°M)	0.01	ν_{21}	0.71	α_{22}(1/K)	$28.4\cdot 10^{-6}$
k_{11}^m(kg$_m$/m s °M)	$4.5\cdot 10^{-9}$	ν_{13}	0.043	α_{33}(1/K)	$4.16\cdot 10^{-6}$
k_{22}^m(kg$_m$/m s °M)	$1.15k_{11}$	ν_{31}	0.52	β_{11}(1/°M)	$36.8\cdot 10^{-4}$
G_{12} (Pa)	$3\cdot 10^8$	ν_{23}	0.073	β_{22}(1/°M)	$18.0\cdot 10^{-4}$
δ (°M/K)	2	ν_{32}	0.45	β_{33}(1/°M)	$1.8\cdot 10^{-4}$

r – heat of the phase change (*toplina isparavanja*); c_m – specific moisture (*specifični sadržaj vode*); k_{jl}^m – mass conduction coefficient (*koeficijent vodljivosti vlage*); G_{ij} – shear modulus (*modul smicanja*); δ – temperature gradient coefficient (*koeficijent temperaturnoga gradijenta*); ν_{ij} – Poisson's ratio (Poissonov koeficijent); α_{ij} – thermal expansion coefficient (*koeficijent* toplinskog širenja); β_{ij} – shrinkage (contraction) coefficient (*koeficijent utezanja*)



a) b) c) d)

Figure 6 Fields of temperature, moisture, stress and displacement at t = 111 h
Slika 6. Polja temperature, koncentracije vlage, naprezanja i pomicanja za t = 111 h

Table 4 Physical properties of wood (beech) (Horman et al, 2003)
Tablica 4. Fizikalna svojstva drva (bukovine) (Horman et al, 2003)

ρ kg/m^3	c J/kg K	k W/m K	E Pa	G Pa	ν -	α 1/K	σ_0 Pa
950	2950	0.54	$4.3 \cdot 10^8$	$1.6 \cdot 10^8$	0.35	$3.2 \cdot 10^{-5}$	$1.2 \cdot 10^6$

ρ – density (*gustoća*); c – specific heat (*specifična toplina*); k – heat conduction coefficient (*koeficijent toplinske vodljivosti*); E – elastic modulus (*modul elastičnosti*); G – shear modulus (*modul smicanja*); ν – Poisson's ratio (*Poissonov koeficijent*); α – coefficient of thermal expansion (*koeficijent toplinskog širenja*) ; σ_0– yield stress (*naprezanje na granici tečenja*)

[redacted]

Figure 7 Physical domain (left), and steam temperature (right)
Slika 7. Fizikalni prikaz (lijevo) i promjena temperature vodene pare (desno)

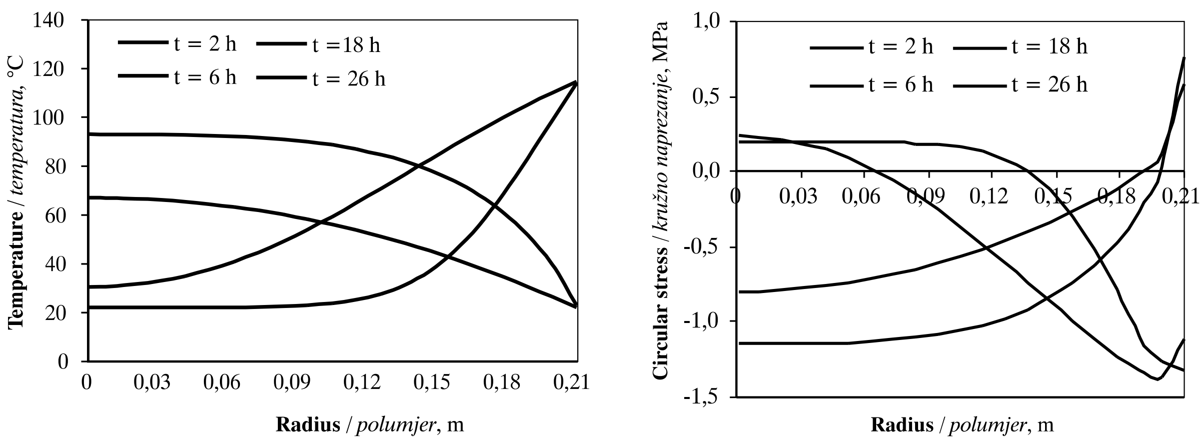


Figure 8 Temperature profile at $\varphi = const.$ (left) and circular stress at $\varphi = const.$ (right)
Slika 8. Profil temperature u $\varphi = const.$ (lijevo) i cirkularno naprezanje u $\varphi = const.$ (desno)

5 CONCLUSIONS
5. ZAKLJUČAK

The numerical method has been outlined for stress analysis and its applicability to the solution of a range of both steady and transient problems involving various elastic, porous, orthotropic and elasto-plastic materials has been demonstrated. The mathematical model and numerical calculation, employing the finite volume method presented, enable the design and construction of a chair and prediction of distribution of deformation and stresses in wood during the process of wood drying and steaming.

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