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Nonlinear mechanics of hyper elastic polyurethane furniture foams*

Nelinearna mehanika hiperelastičnih poliuretanskih pjena za namještaj*

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ABSTRACT • *Upholstered furniture intended to provide better sleep and rest, especially furniture for disabled persons, require careful design of elastic spring systems. In the majority of cases, when designing new articles, both furniture designers and manufacturers rely on long-term experience and craftsman's intuition. On the other hand, the accumulated interdisciplinary knowledge of modern medical laboratories as well as furniture certification offices indicate that it is necessary to carry out investigations related to the mechanical properties of raw materials used to manufacture furniture and to conduct virtual modelling of the phenomena connected with the contact of the human body with the elastic base. The aim of this study was to determine the elastic properties of hyper-plastic polyurethane foams applied in furniture industry, to elaborate mathematical models of these materials on the basis of non-linear Mooney-Rivlin models and to conduct a non-linear numerical analysis of contact strains in a deformed seat made of polyurethane foam. The results of the experiments revealed that the mechanical properties of polyurethane foams are described properly by the Mooney-Rivlin model. Knowing the mechanical properties of these foams, it is possible to create freely complex furniture elastic systems. The state of strains in the contact of the human body with foam depends on the friction between these bodies. Therefore, in practice, it is advisable to design seat systems resulting in minimal frictions between the user's clothes and the furniture seat.*

Key words: *furniture design, hyperplasic polyurethane foam, nonlinear analysis*

SAŽETAK • *Ojastučeni namještaj za spavanje ili odmor, a osobito namještaj za osobe s invaliditetom, zahtijeva vrlo pažljivo dizajniranje sustava elastičnih opruga. Pri dizajniranju novih proizvoda dizajneri i proizvođači svoj rad najčešće temelje na dugogodišnjem iskustvu i intuiciji. Osim toga, akumulirane interdisciplinarne spoznaje modernih medicinskih laboratorija, kao i ustanova za atestiranje namještaja, upozoravaju na potrebu provedbe istraživanja mehaničkih svojstava sirovina koje se upotrebljavaju u proizvodnji namještaja, kao i na potrebu računalnog modeliranja elemenata vezanih za dodir ljudskog tijela s elastičnom podlogom. Cilj ove studije bio je utvrditi elastična svojstva hiperplastičnih poliuretanskih pjena koje se primjenjuju u industriji namještaja, razraditi matematičke modele navedenih materijala na temelju nelinearnih Mooney-Rivlin modela i provesti nelinearnu numeričku analizu naprezanja u deformiranom sjedištu od poliuretanske pjene. Rezultati eksperimenata otkrili su da su mehanička svojstva poliuretanskih pjena pravilno opisana Mooney-Rivlin modelom. Poznavanjem mehaničkih svojstava navedenih pjena moguće je slobodno kreirati složene elastične sustave namještaja. Uvjeti napre-*

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zanja pri dodiru ljudskog tijela s pjenom ovise o razini trenja među navedenim tijelima. Stoga je u praksi preporučljivo dizajnirati sustave sjedalica s minimalnim trenjem između korisnikove odjeće i sjedalice.

Ključne riječi: *dizajn namještaja, hiperplastična poliuretanska pjena, nelinearna analiza*

1 INTRODUCTION AND OBJECTIVE OF STUDY
1. UVOD I CILJ STUDIJE

From the point of view of human physiology, sitting for several hours is unfavourable for both the nervous and muscle-skeletal systems. Although maintaining this position is less tiresome when compared to the standing position, it can cause much higher loads (by about 40%) in the lumbar spine. Also, the incorrect distribution of the body weight on the seat may lead to point loads of the cardiovascular system (Fig.1). Consequently, improper, long-term positioning of the body on the ill-designed base often causes pain complaints, degenerative changes of joints, thrombi as well as surface inflammations of the vascular system of lower limbs.

It is easy to see that the sitting position has become dominant in our modern style of life. Even passive relaxation, as a rule, takes place on furniture designed for sitting and relaxation which leads not only to psychological but also to physiological relaxation.

The first step in trying to solve the problem of uncomfortable seats can be the analysis of the man-seat system and, in particular, the analysis of the mechanical properties of polyurethane foams and numerical modelling of complex layer systems.

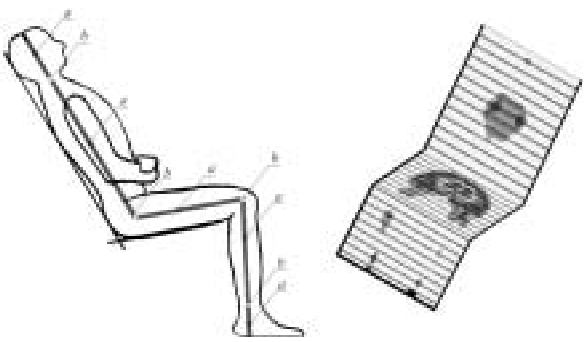


Figure 1 Man-seat system: a) centres of gravity of individual body parts, b) distribution of stresses on the seat surface
Slika 1. Sustav čovjek – namještaj za sjedenje: a) središta gravitacije pojedinih dijelova tijela, b) raspodjela naprezanja na površini pri sjedenju

The objective of the investigations was to determine the elastic properties of hyperplastic polyurethane foams applied in the furniture industry, to elaborate mathematical models of these materials on the basis of non-linear Mooney-Rivlin models and to conduct a non-linear numerical analysis of contact strains in a deformed seat.

2 MATERIAL AND METHODS
2. MATERIJAL I METODE

The following four polyurethane foams: T2516, T2838, T3530 and T4060, most frequently used in production, were selected for laboratory investigations. Using ordinary commercial polyurethane foam blocks, five samples were cut out from each type of foam in the shape of cubes with the sides of 100 mm (Fig. 2). A rectangular network of 10 x 10 mm module was plotted on the face surface of each sample with the aim of carrying out a detailed analysis of the form and size of deformations. The experimental samples were placed between pressure blocks of the ZWICK 1445 testing machine and subjected to compression with the velocity of 100 mm/min. The deformations of the examined samples were recorded both on the testing machine and with the help of a digital image correlation (DIC) system, a digital camera connected to a computer which recorded the form of deformations at 25, 50 and 75% deformation. The camera was controlled by software in the LabView environment developed by the National Instruments Company. Using the obtained laboratory results, strain-deformation correlations were elaborated for each type of experimental foam.

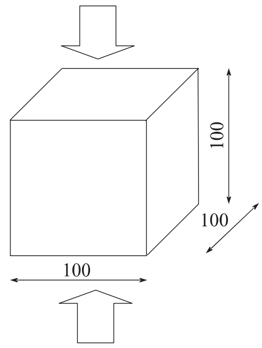


Figure 2 Scheme of the uniaxial foam compression
Slika 2. Prikaz jednoosnog tlačenja pjene

Foam is a body made up of many network-forming cells whose walls form polygonal plates. A typical foam strain-deformation diagram (Fig. 3) consists of three stages. During the first stage, which accounts for 5% of the entire deformation, bending of cell walls takes place and deformations are linear. In the course of the second stage, cell walls, like thin tubes or plates, lose their stability causing large deformations. During this stage the air is also removed from intercellular spaces. The third stage is characterised by the compression of the deformed cell walls and a considerable increase of rigidity. Table 1 presents modules of linear elasticity E_1, E_2 and E_3, which were determined for each stage of compression.

When building the mathematical model of the elastic polyurethane foam, the authors assumed that it was a model:
- Of isotropic and non-linear material,
- Consisting of evenly distributed cells capable of large deformations,
- Capable of large deformations (above 90%) during compression,

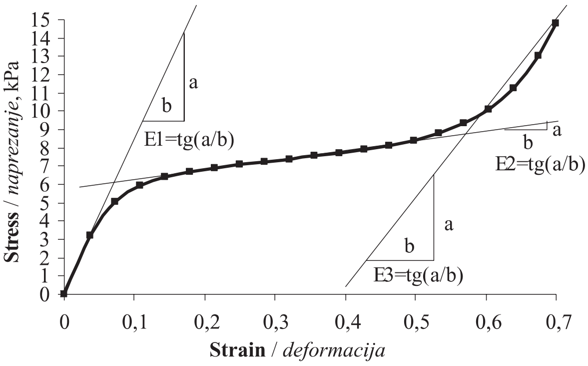


Figure 3 Scheme of determination of rigidity moduli of polyurethane foams at different deformation ranges
Slika 3. Dijagram utvrđivanja modula krutosti poliuretanskih pjena prema različitim rasponima deformiranosti

– Which required geometrical non-linearity during consecutive stages of deformation analysis.

Hence, strains in the function of tension energy for foam can be expressed in the following form:

$$\sigma_i = \frac{\partial W}{\partial L_i}.$$

The compression energy is expressed as follows:

$$W = f(I_1, I_2, I_3),$$

where:

$$I_1 = L_1^2 + L_2^2 + L_3^2,$$
$$I_2 = L_1^2 L_2^2 + L_2^2 L_3^2 + L_3^2 L_1^2,$$
$$I_3 = L_1^2 L_2^2 L_3^2.$$

The strain function for axial compression assumes the following form:

$$\sigma L = \left(L^2 - \frac{1}{L}\right)\left[2\left(\frac{\partial W}{\partial I_2}\right) + \frac{2}{L}\left(\frac{\partial W}{\partial I_1}\right)\right].$$

Therefore, the Mooney-Rivlin equation, appropriate for hyperplastic materials (of large deformations of up to 200%), can be presented as follows:

$$W = (I_1, I_2) = C_1(I_1 - 3) + C_2(I_2 - 3).$$

This equation assumes the following form for a uniaxial compression or tension:

$$\sigma = 2\left(C_1 + \frac{C_2}{L}\right)\left(L - \frac{1}{L^2}\right).$$

When this equation is transformed to the form:

$$\frac{\sigma}{2\left(L - \frac{1}{L^2}\right)} = \frac{1}{L}C_2 + C_1,$$

the equation of a straight line is obtained, to be used to determine C_1 and C_2 correlations, which are essential for further numerical analysis:

$$y = ax + b$$

where:

$$y = \frac{\sigma}{2\left(L - \frac{1}{L^2}\right)}, \; a = \frac{1}{L_1}.$$

The next stage of investigation involved the development of numerical models of hyperplastic polyurethane foams in the ABAQUS (1) system environment and the comparison of compatibility of the calculated strains and deformations with the results of the laboratory experiments in the axial compression test. This served as the basis for the elaboration of proper numerical models of the man-seat system, also in the ABAQUS system, presenting the mattress in the form of a grid, 100 x 500 mm in size, and a human body in the form of an analytical curve with the radius of 86 mm, which corresponds to the dimensions of a man's thigh of the European population with the 95 percentile (Fig. 4).

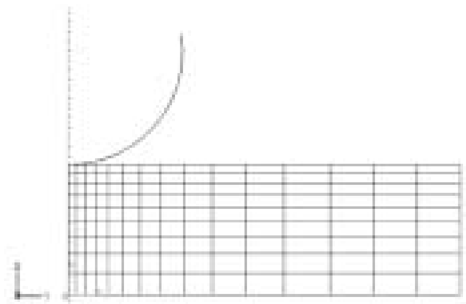


Figure 4 Numerical model of a part of human body loading the mattress
Slika 4. Numerički model dijela ljudskog tijela koji opterećuje ležaj

The described model applied the C_1 and C_2 coefficients determined earlier for each foam and, in addition, assumed two values of the friction coefficient - f_1 = 0.8 and f_2 = 0.4 - between the human body and the base. This served as the basis for the assessment of effectiveness of the examined foams for the construction of human factor engineering and functional seats of furniture for rest.

3 RESEARCH RESULTS AND ANALYSIS
3. REZULTATI ISTRAŽIVANJA I ANALIZA

Figure 5 presents the strain-deformation correlation for each type of the experimental foam. It is quite clear that T2838 and T4060 foams had the highest rigidity, while T2516 and T 3530 foams were much softer. Initially, therefore, it can be concluded that T2516 and T 3530 foams should be applied as the external layers of the mattress, i.e. those which remain in a direct contact with the user's body, whereas T2838 and T4060 foams should be employed as internal layers, which prevent significant displacements, especially in the case of high body weight.

Table 1 also shows clearly that polyurethane foams are characterised by the changing value of the Young modulus at different stages of compression. In the case of T2516 and T 3530 foams, the E_3 to E_1 ratio = 0.76 – 0.82, whereas the E_3 to E_1 ratio for the T2838 and T4060 foams ranges from 0.43 to 0.58. The above-described significant differences allow for a greater freedom in the selection of foam rigidity during modelling of complex systems of multi-layer mattresses.

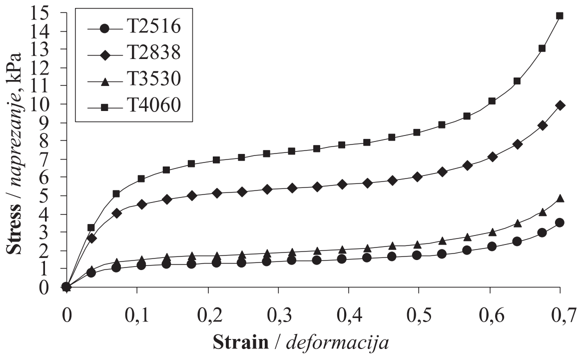


Figure 5 Rigidity of polyurethane foams in the stress deformation system
Slika 5. Krutost poliuretanskih pjena u sustavu naprezanja i deformacije

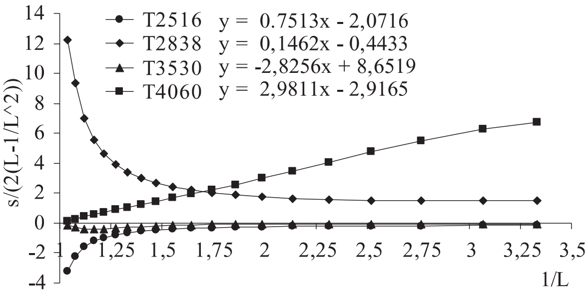


Figure 6 Scheme of determination of C_1 and C_2 constants in Mooney-Rivlin equations
Slika 6. Dijagram utvrđivanja konstanti C_1 i C_2 u Mooney-Rivlin jednadžbama

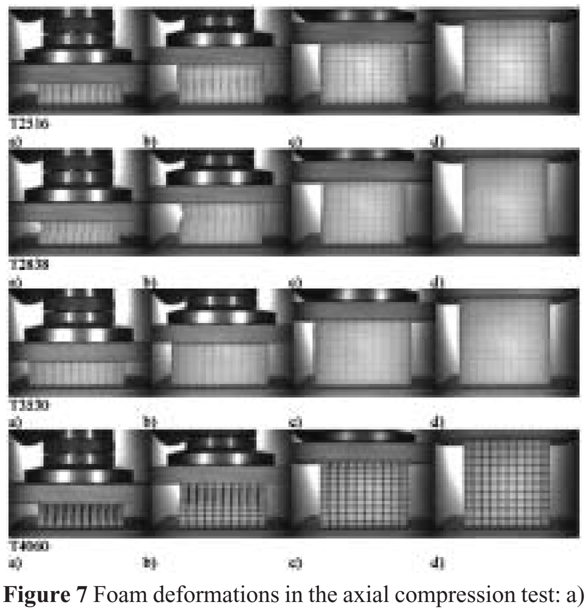


Figure 7 Foam deformations in the axial compression test: a) 70% deformation, b) 50% deformation, c) 25% deformation, d) the state before deformation
Slika 7. Deformacije pjene u testu tlačnog opterećenja: a) 70-postotne deformacije, b) 50-postotne deformacije, c) 25-postotne deformacije, d) stanje prije deformacije

Material constants in the Mooney-Rivlin equation were determined on the basis of Figure 6 and later presented in Table 2. Data collected in this way were used to develop a numerical model.

When developing mathematical models of the examined foams, the form of the deformation in the axial compression test was determined in the first place (Fig. 7). It is clearly shown that, irrespective of the degree of loading, foams lose their shape without bulging. Their side walls remain straight or slightly concave, which distinguishes them from such elastic materials as gum or rubber.

Table 3 presents values of normal strains, which were also calculated for individual stages of loading.

Comparison of values of these strains with the results of the numerical calculations (Fig. 8) makes it possible to observe that, at 70% of the initial deformation, the strain values determined in the samples during numerical calculations correspond precisely to the strain values determined during the laboratory experiment. This appears to suggest that numerical models based on experimental models and use of Mooney-Rivlin equations describe quite well the behaviour of polyurethane foams

Table 1 Modulus of linear rigidity of polyurethane foams
Tablica 1. Modul linearne krutosti poliuretanskih pjena

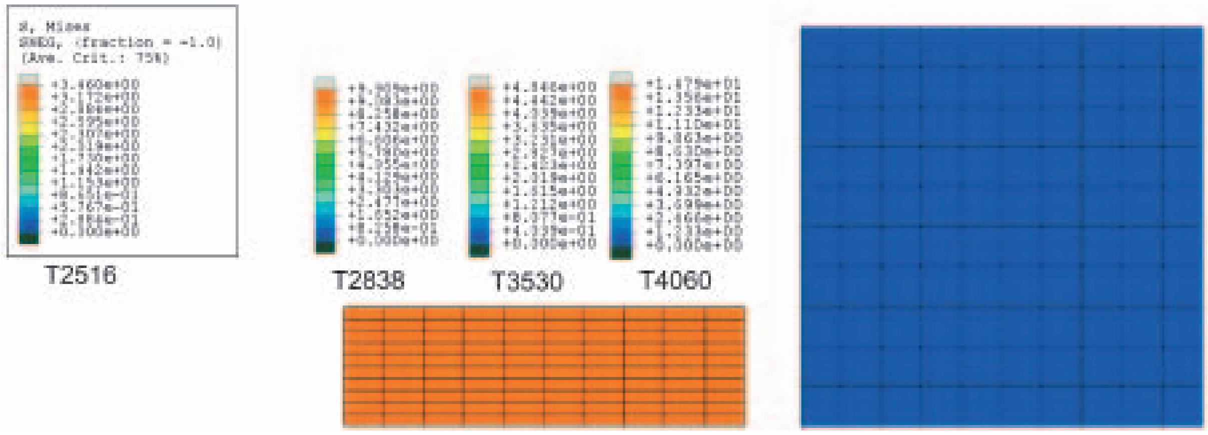
Type of foam *Vrsta pjene*	Young's moduli, kPa *Youngov modul, kPa*		
	E_1	E_2	E_3
T2516	14,44	1,52	11,06
T2838	56,12	3,75	24,30
T3530	18,69	2,32	15,50
T4060	70,25	6,22	40,82

Table 2 Values of C_1 and C_2 constants in Mooney-Rivlin equations
Tablica 2. Vrijednosti konstanti C_1 i C_2 u Mooney-Rivlin jednadžbama

Type of foam *Vrsta pjene*	Coefficient *Koeficijent*	
	C_1	C_2
T2516	-2,0716	0,7513
T2838	-0,4433	0,1462
T3530	8,6519	-2,8256
T4060	-2,9165	2,9811

Table 3 Normal strains in polyurethane foams at different deformation values
Tablica 3. Normalna naprezanja poliuretanskih pjena prema različitim vrijednostima deformacija

Type of foam *Vrsta pjene*	Strain *Deformacija*		
	0,25	0,50	0,70
	Stress, kPa *Naprezanje, kPa*		
T2516	1,33	1,70	3,46
T2838	5,23	6,04	9,91
T3530	1,79	2,36	4,85
T4060	7,07	8,42	14,81



a) b)

Figure 8 Numerical model of the examined foams and strain values of von Mises: a) the state after deformation, b) the state before deformation

Slika 8. Numerički model ispitanih pjena i vrijednosti naprezanja prema von Misesu: a) stanje nakon deformacije, b) stanje prije deformacije

during compression. Furthermore, it is worth noticing that the deformation form presented in Figure 8 is also similar to the form of deformations shown in Figure 7.

In order to find the answer to the question which of the examined foams is most suitable for human factor engineering seats and mattresses, the authors carried out numerical calculations with the help of the ABAQUS system, which simulates the pressure of the human thigh on the surface of a mattress manufactured from one of the foams used in the investigations. The results of these calculations are presented in Figures 9 and 10.

A number of interesting observations can be made when analysing the above-presented distribution of strains according to Mises at different friction coefficients between the user's body and the mattress. T2516 and T3530 foams favour concentration of strains in the neighbourhood of ischia and support the user's body non-uniformly, whereas T2838 and T4060 foams tran-

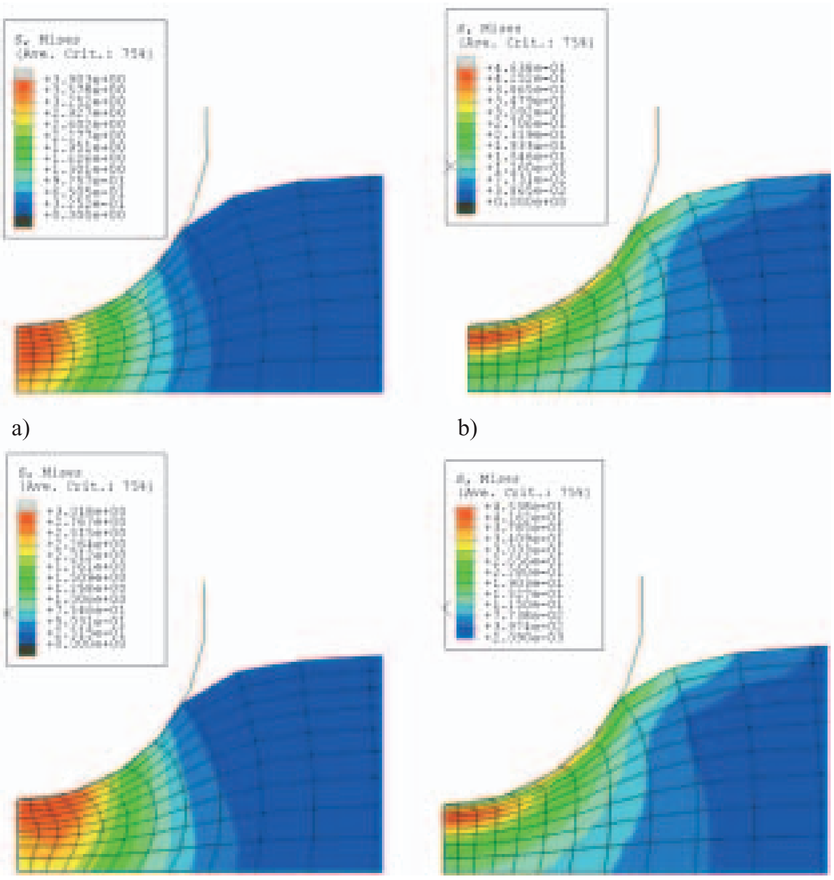


Figure 9 Distribution of von Mises strains in foams, caused by the operational loading at the friction coefficient of 0.8: a) T2516; b) T2838; c) T3530 and d) T4060

Slika 9. Raspodjela naprezanja u pjenama, prema von Misesu kao rezultat stvarnog opterećenja uz koeficijent trenja 0,8: a) T2516; b) T2838; c) T3530 i d) T4060

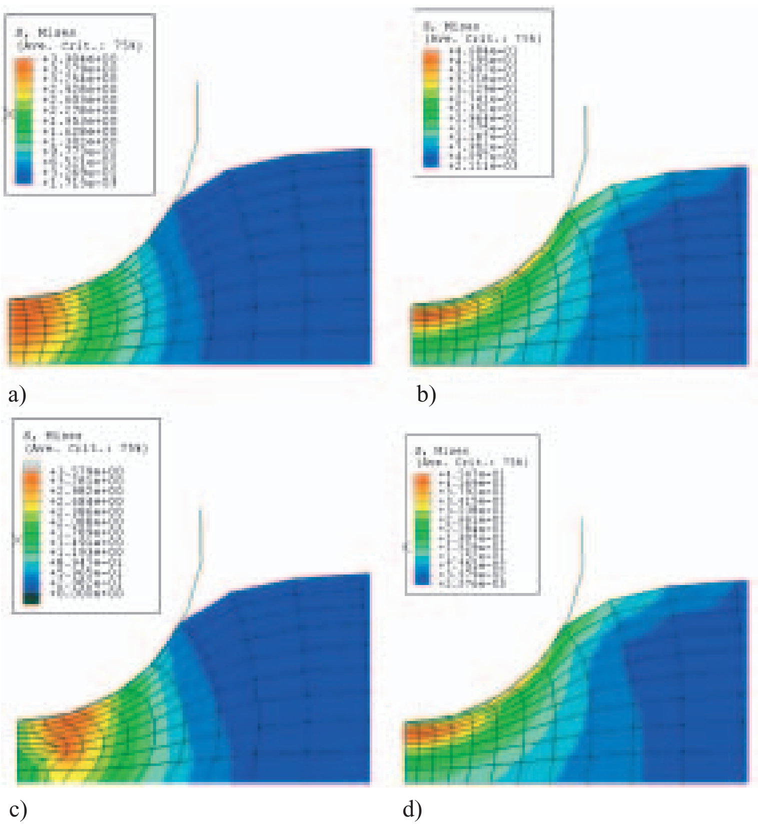


Figure 10 Distribution of von Mises strains in foams, caused by the operational loading at the friction coefficient of 0.4: a) T2516; b) T2838; c) T3530 and d) T4060

Slika 10. Raspodjela naprezanja u pjenama, prema von Misesu kao rezultat stvarnog opterećenja uz koeficijent trenja od 0,4: a) T2516; b) T2838; c) T3530 i d) T4060

sfer pressures exerted by the user's body more uniformly and ensure higher and fuller comfort resulting from the response of the base. These observations prompt further investigations on the selection of optimal, multi-layer systems manufactured from polyurethane foams in which both interesting mechanical properties will be used. The impact of friction on the value of the occurring strains and the form of their distribution is important and deserves separate, detailed numerical analysis.

4 CONCLUSIONS
4. ZAKLJUČCI

Based on laboratory investigations and results of numerical calculations the following conclusions can be drawn:

1. T-type polyurethane foams are isotropic bodies of nonlinear characteristics,
2. Mooney-Rivlin model describes properly the behaviour of T-type foams,
3. Numerical solutions that use models of hyperplastic bodies make it possible to perform a precise analysis of the phenomenon of contact between the human body and the elastic mattress,
4. Friction exerts a significant influence on the value and form of distribution of contact strains.

5 REFERENCES
5. LITERATURA

1. ABAQUS/Standard, ABAQUS/Explicit, ABAQUS/CAE, Hyperelastic behavior, Helps.
2. Grbac, I. 2005: Krevet i zdravlje. Sveučilišni udžbenik. Šumarski fakultet, Akademija šumarskih znanosti, 271 s.
3. Grbac, I.; Ivelić, Ž. 2005: Ojastučeni namještaj. Sveučilišni udžbenik. Šumarski fakultet, Akademija šumarskih znanosti, 293 s.
4. Hirota, G. 2002: An Improved Finite Element Contact Model for Anatomical Simulations. A Dissertation at the faculty of The University of North Carolina in the Department of Computer Science, 147 p.
5. Klute, G. K.; Hannaford, B. 2000: Accounting for Elastic Energy Storage in McKibben Artificial Muscle Actuators. ASME Journal of Dynamic Systems, Measurement, and Control, 122(2):386-388,
6. Misra, S.; Okamura, A.M.; Ramesh, K. T. 2006: Force Feedback is Noticeably Different for Linear versus Nonlinear Elastic. Department of Mechanical Engineering, The Johns Hopkins University. Baltimore, MD 21218 USA

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